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Comments on "A Minimization Technique for TANT Networks"

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Abstract—Some comments on a recent note for the synthesis of TANT networks are presented. Counter examples that show some defects of the technique are also included.

Index Terms—Consensus operation, essential prime implicants, gate cost criterion, minimal TANT expression, prime implicants, TANT networks.

In the above short note,[1] a technique that generates a minimal-gate TANT realization of a given function is presented. The authors want to point out three points which are related to the technique.

First, the definition of essential prime implicant in the Quine–McCluskey procedure[2] is different from Koh's definition. For example, in Koh's first example, the EPI $wx'z'$ and $yx'z'$ do not satisfy the definition for essential prime implicants in the Quine–McCluskey procedure. The set of all essential prime implicants does not necessarily cover the function.

Second, an arbitrary selection of the starting set of prime implicants (EPI in Koh's note) will not necessarily lead to a minimal TANT network. We show this in Example 1.

Example 1: Realize $f=\Sigma\,(0, 1, 2, 6, 7, 8, 11, 12, 14)$. This function f to be realized is the same function in the second example by Koh. Koh picked his set of prime implicants (PI) as shown in Fig. 1. However, it is also possible to pick a different set of PI as shown in Fig. 2. of the set in Fig. 2 was picked, the network resulting from Koh's technique would be ten gates instead of nine gates.

Finally, the technique uses "useful prime implicants" together with the set of EPI to generate the "minimal TANT expression." However, it is shown in Example 2 that "useful prime implicants" are not necessarily desirable.

Example 2: Realize $f=\Sigma\,(0, 6, 10, 11, 14)$.

$$\text{EPI: } \left.\begin{array}{l} (A)\ w'x'y'z' \\ (B)\ xyz' \\ (C)\ wyx' \end{array}\right\} \text{ which covers } f.$$

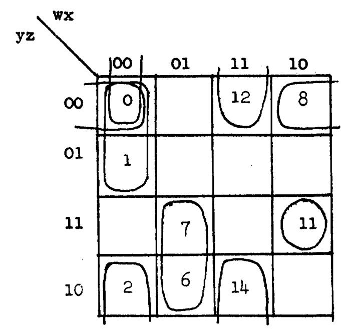

Fig. 1. One way of covering $f=\Sigma\,(0, 1, 2, 6, 7, 8, 11\ 12, 14)$.

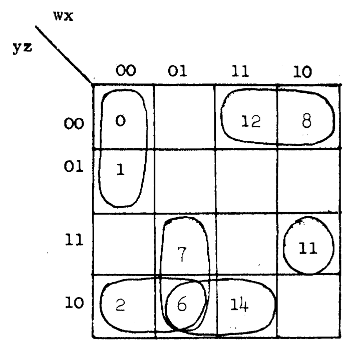

Fig. 2. Second way of covering $f=\Sigma\,(0, 1, 2, 6, 7, 8, 11, 12, 14)$.

A useful prime implicant is (D) wyz'. (D) is useful because it has the same head as (C).

By Koh's technique, the minimal TANT expression for f using (A), (B), (C), and (D) is

$$f = w'x'y'z' + xyz' + wy(xz)'$$

or

$$w'x'y'z' + xy(xz)' + wy(xz)'.$$

Both need nine gates. But by excluding the "useful prime implicant" (D),

$$\begin{aligned} f &= (A) + (B) + (C) \\ &= w'x'y'z' + xyz' + wyx' \end{aligned}$$

which needs only eight gates.

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